Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NUMBER THEORY.

255. Proposed by FRANK IRWIN, University of California.

Given any arithmetical progression whose first term a and common difference d are relatively prime integers, and any finite set of positive integers $m_1, m_2, \cdots$ also relatively prime to d, it is required to determine an integer n such that the multiples of $m_1, m_2, \cdots$ may occupy the same positions in the series of natural numbers beginning with n as they do in the arithmetical progression. This is to say that if the kth, the $(m_1 + k)$th, the $(2m_1 + k)$th, $\cdots$ terms of the progression are divisible by m_1, so also will be the kth, the $(m_1 + k)$th, the $(2m_1 + k)$th, $\cdots$ terms of the series $n, n + 1, n + 2, \cdots$, etc. Show that n may be determined as the solution of a congruence $An + B \equiv 0 \pmod{C}$, whose coefficients, A, B, are constants independent of the number and value of the m's.

256. Proposed by FRANK IRWIN, University of California.

Let p be an odd prime, and let the notation $1/k$ stand for the solution of $kx \equiv 1 \pmod{p}$. Then show that if the sum of the numbers

$$1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{(p-1)/2}$$

be congruent to zero (mod p)—should that be possible—the same is true for the sum of their products two at a time, as well as four at a time.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

A solution of 450 was received from G. Y. SOSNOW and one of 454 from J. J. GINSBURG which have not been acknowledged.

458. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that n terms of the series $1 + 3 + 4 + 6 + 7 + 9 + 10 + \cdots$ is $\frac{1}{4}(n + 1)(3n - 1)$ when n is odd, and $\frac{n}{2}\left(\frac{3n}{2} + 1\right)$ when n is even.

SOLUTION BY GEO. W. HARTWELL, Hamline University.

This series is a combination of two arithmetical progressions $3 + 6 + 9 + \cdots$ and $1 + 4 + 7 + 10 + \cdots$.

When n is even, the first becomes

$$3 + 6 + 9 + \cdots + \frac{3}{2}n = \frac{n}{4}\left(\frac{3}{2}n + 3\right)$$

and the second,

$$1 + 4 + 7 + 10 + \cdots + \left(\frac{3}{2}n - 2\right) = \frac{n}{4}\left(\frac{3}{2}n - 1\right).$$

The sum of these two is

$$\frac{n}{4}\left(\frac{3}{2}n + 3\right) + \frac{n}{4}\left(\frac{3}{2}n - 1\right) = \frac{n}{2}\left(\frac{3}{2}n + 1\right).$$

When n is odd, the first becomes

$$3 + 6 + 9 + \cdots + \frac{3}{2}(n - 1) = \frac{n-1}{4}\left(\frac{3n}{2} + \frac{3}{2}\right)$$

and the second,

$$1 + 4 + 7 + 10 + \cdots + \left(\frac{3n}{2} - \frac{1}{2}\right) = \frac{n+1}{4}\left(\frac{3n}{2} + \frac{1}{2}\right).$$

The sum of these two is

$$\frac{n-1}{4}\left(\frac{3(n+1)}{2}\right) + \frac{n+1}{4}\left(\frac{3n+1}{2}\right) = \frac{1}{4}\left(\frac{3n^2}{2} - \frac{3}{2} + \frac{3n^2}{2} + 2n + \frac{1}{2}\right)$$

$$= \frac{1}{4}(3n^2 + 2n - 1) = \frac{1}{4}(n + 1)(3n - 1).$$

Also solved by L. A. H. Warren, W. C. Eells, George Paaswell, E. H. Worthington, Horace Olson, G. L. Wagar, H. N. Carleton, E. J. Oglesby, H. C. Feemster, Benj. Sinitsscy, W. J. Thome, C. C. Yen, R. M. Mathews, E. F. Canady, N. P. Pandya, J. J. Ginsburg, O. S. Adams, A. M. Harding, and Paul Capron.

GEOMETRY.

485. Proposed by NATHAN ALTSHILLER, University of Colorado.

Find the surface generated by the orthogonal projection of a given line upon a variable plane turning about a fixed axis.

I. Solution by Elijah Swift, University of Vermont.

Take the fixed axis as X-axis, and the common perpendicular of this line and the given line as Z-axis. Call the angle θ made by the line we are projecting and the X-axis, so that the given line has the direction cosines $\cos\theta$, $\sin\theta$, 0; let the coördinates of the point where this line intersects the Z-axis be $(0, 0, a)$. Then the equation of the variable plane may be written in the form

(A) $y - \lambda z = 0$, where λ is a parameter.

The equation of any plane through the given line may be written in the form

(B) $\sin\theta x - \cos\theta y + k(z - a) = 0$, where k is an arbitrary constant.

If (B) is perpendicular to (A), k must equal $-\cos\theta/\lambda$. Substituting this value for k in (B), (A) and (B) give the required equation in parameter form. Eliminating λ, we obtain the explicit equation

$$\tan\theta \cdot xy = y^2 + z^2 - az.$$

Rotating the axes through an angle $\theta/2$ about the Z-axis, and changing the origin to the point $(0, 0, a/2)$, the equation finally becomes

$$x^2\left(\frac{1 - \cos\theta}{2\cos\theta}\right) - y^2\left(\frac{1 + \cos\theta}{2\cos\theta}\right) - z^2 = -\frac{a^2}{4},$$

which is the equation of a hyperboloid of one sheet.

II. Solution by the Proposer.

The orthogonal projection u of a given line p upon a plane β passing through a given axis q, may be obtained as the intersection of β with the plane α perpendicular to β and passing through p. When the plane β turns about q, the projecting plane α turns about p; to each position of β corresponds one and only one position of α, and vice versa. The line u is, therefore, the intersection of two corresponding planes α, β of two projective pencils (p), (q).

I. The lines p, q are coplanar. *The line u generates a cone of second degree (C), of which p, q are elements.*

The tangent planes to (C) along p and q are the planes perpendicular to the plane (pq) and passing through the lines p and q respectively. The line of intersection of these tangent planes is perpendicular to (pq) and is the polar ray of this plane with respect to (C). Hence: *The plane of the two given lines is a plane of symmetry of the cone.*

The orthogonal projection P' of any point P of p upon the plane β lies on the line u and in the plane π through P perpendicular to q. When β varies, P' describes, in the plane π, a circle having for diameter the segment joining P to the point of intersection of π with q. This circle is the curve of intersection of (C) with the plane π. Similarly for the orthogonal projection Q' of any point Q of q upon α. Hence: *The planes perpendicular to the given lines are the planes of the circular sections of the cone.*

This cone is sometimes called the *Orthogonal* cone (Theodor Reye, *Geometrie der Lage*, part I, p. 119, fifth edition).

If the given lines p, q are parallel, the cone becomes a cylinder of revolution.

II. The lines p, q are skew. *The line u generates a hyperboloid of one sheet (H), of which p, q are two rays of the same system.*

The planes perpendicular to the given lines are the planes of circular sections of the hyperboloid. Same proof as for the cone above.